Math 107: Calculus II, Spring 2014: Midterm Exam II
Monday, April 13 2015

Give your **name**, TA and section number:

Name:

TA:

Section number:

1. There are 5 questions for a total of 100 points. The value of each part of each question is stated.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use phones, calculators, books, notes or any other paper. Write all your answers on this booklet. If you need more space, you can use the back of the pages.
4. **Unless specified otherwise, you must show ALL your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Make sure you understand what each question asks of you.

Please read the following statement and then sign and date it:

"I agree to complete this exam without unauthorized assistance from any person, materials, or device."

Signature:

Date:

Question	Points	Score
1	30	
2	20	
3	12	
4	18	
5	20	
Total:	100	

1. Let X be a discrete random variable with range $\{0,\ 1,\ 2,\ 3,\ 4\}$ and probability mass function

$$P(X=0)=\frac{1}{9},\ \ P(X=1)=\frac{2}{9},\ \ P(X=2)=\frac{3}{9},\ \ P(X=3)=\frac{2}{9},\ \ P(X=4)=\frac{1}{9}.$$

For the following questions, carry out the computations and leave your answer as an irreducible fraction.

(a) (12 points) Find $E(X)$ and $\mathrm{var}(X)$. **Show all your work!**

(b) (8 points) Find $P(1 \le X \le 3)$. **Show all your work!**

(c) (10 points) We measure X four times independently. What is the probability that $X = 2$ for exactly two of the four measurements? **Show all your work!**

2. Suppose that a continuous random variable X has distribution function

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0; \\ 0 & \text{otherwise.} \end{cases}$$

(a) (5 points) Find $P(X \leq 5)$. **Show all your work!**

(b) (15 points) Find E(X). **Show all your work!**

3. (12 points) The following pictures, in some order, denote the image of the vector $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ under the transformations given by the matrices.

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad A_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \quad A_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad A_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Match the pictures with the corresponding matrix by writing A_i for the correct value of i in the line next to the picture. **You do not have to show any work.**

(a) (a) ____________

(b) (b) ____________

(c) (c) ____________

(d) (d) ____________

4. (18 points) Find the general solution to the differential equation

$$\frac{dy}{dx} = y(1-y).$$

Show all your work! You will be graded on the completeness and quality of your solution.

5. Let $A = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

(a) (5 points) State the definition of what it means for a scalar λ to be an eigenvalue of A.

(b) (15 points) Find the eigenvalues of A (start from the definition, carry out and explain all the steps in your argument.)
Show all your work! You will be graded on the completeness and quality of your solution.